An Analytical Model of Multihop Connectivity of Inter-Vehicle Communication Systems

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Abstract

Taking advantage of the proliferation of wireless communication devices, we could well develop Advanced Transportation Information Systems based on Inter-Vehicle Communication (IVC), in which drivers can have faster response to incidents and are able to communicate critical information in wake of disasters. Whether such IVC systems are feasible or not is highly related to the performance of multihop connectivity. Existing analytical studies of multihop connectivity, however, usually assume Poisson distribution of communication nodes or uniform distribution of vehicles on a road, and simulation-based studies are not suitable for real-time applications with computationally costly traffic simulators. In this paper, we present an analytical model for multihop connectivity of IVC in a traffic stream, in which positions of vehicles are

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all known through observations, traffic simulators, or traffic theories. After introducing Most-Forwarded-within-Range communication chains and node- and hope-related events, we derive a recursive model of node and hop probabilities and further define a number of performance measures of multihop connectivity. We then apply the model to study multihop connectivity of IVC in both uniform and non-uniform traffic and obtain results consistent with those in literature. The new analytical model is efficient without repeating traffic simulations while capable of capturing the impact of arbitrary distribution patterns of vehicles. Thus it is suitable for evaluating connectivity of IVC for different traffic congestion patterns and extended for studies of other situations.

1 Introduction

As wireless communication devices proliferate, Advanced Transportation Information Systems based on Inter-Vehicle Communication (IVC) (Recker et al., 2007) are not only possible, but probable. Different from existing centralized transportation information systems (e.g., Boyce et al., 1994), such systems impose no capital investment on transportation agencies and can evolve into full-fledged systems gradually like the Internet. They can be used as incident response systems, are resilient to disasters such as earthquakes, and can be anchored to the Internet.

In an IVC system, vehicles equipped with wireless devices form a mobile ad hoc network. Since vehicles usually move at high speeds and are constantly entering and leaving roadway segments, the communication topologies in an IVC system are dynamic (Rudack et al., 2002). IVC can be coupled with vehicle movement when, for example, a vehicle would like to send a piece of message to an upstream vehicle but there exists no communication path between them. In such a case, the information source can "carry" the message until meeting an equipped vehicle traveling in the opposite direction, who then will be able to relay the message when it meets the information receiver. Such information relay through vehicle movement obviously becomes less necessary when either the proportion of IVC-equipped vehicles (*market penetration rate*) and the density of vehicles is relatively high, or the *transmission range* of the communication device is sufficiently large.

As in other mobile ad hoc networks, multihop connectivity is a fundamental technology issue in IVC systems, together with routing protocols (Perkins, 2000), network capacity (Gupta and Kumar, 2000), and optimal throughput (Takagi and Kleinrock, 1984). Estimating connectivity a priori is important to help guide specification of appropriate communication devices, routing protocols, database management schemes, and the range of effective applications. Multihop connectivity has been studied for both stationary and mobile communication networks since the development of a percolation theory in (Gilbert, 1961). Studies include communication networks that are both one-dimensional (Cheng and Robertazzi, 1989; Piret, 1991) and two-dimensional (Philips et al., 1989); and networks, in which the number of nodes can be finite (Desai and Manjunath, 2002) or infinite (asymptotic) (Gupta and Kumar, 1998). Research methodologies include theoretical analyses (Dousse et al., 2002) as well as Monte Carlo simulations (Tang et al., 2003); performance measures of connectivity include expected propagation distance (Cheng and Robertazzi, 1989), the probability for having at least one communication path between two nodes (Hartenstein et al., 2001), the k-connectivity (Penrose, 1999; Bettstetter, 2002), or the critical transmission range for asymptotic cases (Philips et al., 1989; Piret, 1991; Gupta and Kumar, 1998).

It has been standard in all of these studies to assume that communication nodes follow a spatial Poisson distribution on a line or plane (Cheng and Robertazzi, 1989; Wang, 2007); i.e., their locations are independent of each other, and their density is uniformly random at any location. However, it has been noted in (Cheng and Robertazzi, 1989) that this assumption is not valid for a traffic stream due to interactions between vehicles' movements. Indeed, such a spatial Poisson distribution is often violated in network vehicular traffic (Jin, 2003). First, positions of vehicles are not independent, since drivers have to observe carfollowing or lane-changing rules. Second, the density of vehicles can vary dramatically along a roadway due both to driving behavior as well as to restrictions of network geometry. For example, around a lane-drop or merging area, traffic density is usually significantly higher in the upstream section; when a shock wave forms (Lighthill and Whitham, 1955), traffic density is higher in the downstream part; vehicles tend to form clusters in sparse traffic; and traffic signals can cause gaps between vehicle platoons. Therefore, it is important to include such characteristics of network vehicular traffic when studying multihop connectivity of IVC.

In (Hartenstein et al., 2001; Yang and Recker, 2005), multihop connectivity of IVC is statistically studied by repeated simulations with a vehicular traffic simulator. Using simulation, the vehicular traffic pattern can be realistically captured, but the high computational cost prevents its use for analyzing a large-scale road network. In this paper, we develop an analytical model of multihop connectivity of an IVC system, which is efficient in the sense of not requiring repeating traffic simulations while capable of capturing the impact of arbitrary distribution patterns of vehicles. Since vehicular traffic patterns yield extra complexities to an IVC system, we constrain our analysis to a one-dimensional traffic stream on a link, omitting any effect of merging and diverging vehicles on information propagation. Such a 1-D model is not prohibitively restrictive, since communication ranges provided by current technologies (such as 802.11 protocols) are usually much smaller than a section of roadway, and buildings and other objects in urban areas can block such short-range communications between two vehicles on different streets. However, this study could be a springboard for further investigation of information propagation on a network of roads.

The remainder of the paper is organized as follows. In Section 2, we lay out the conceptual framework and corresponding definitions of our model. In Section 3, we derive a recursive model of end node probabilities, based on which we define a number of performance measures of multihop connectivity in an IVC system. In Section 4, we study examples of both uniform and non-uniform traffic. In Section 5, we present a discussion on possible implications of this study.

2 Conceptual framework

2.1 Assumptions

An IVC system is complicated with coupled vehicular dynamics and information propagation. In our study, we make a major simplification assumption that information propagation is *instantaneous* with respect to vehicle movement. Such an assumption is supported by experiments in (Briesemeister et al., 2000), in which it takes 110 ms to transmit a message of 73 bytes with a device whose transmission rate is 3.6 kb/s. If the transmission range is 500 m, then a message can propagate as far as 5000 m for 10 hops, or 1.1 s. During such a short time, the change in the relative distance between two vehicles traveling in opposite directions with a free flow speed of 65 mile/hr is about 58 m. Thus, under such conditions, vehicle movement is inconsequential compared to information propagation. However, the assumption of instantaneous information propagation is only meaningful when equipped vehicles are well connected and the transmission time is short; i.e., when we have relatively high market penetration rates, long transmission ranges, short messages, large communication bandwidths, and efficient communication protocols. Among all of these requirements, those on communication hardware and software can be satisfied with current technologies; the required market penetration rate of wireless communication devices for the assumption is not likely during the initial deployment of an IVC system. Under the assumption of instantaneity, an IVC system is equivalent to a stationary communication network, in which vehicles' positions are deterministic and can be predicted by either traffic flow models or simulation (Jin, 2003). Note that, however, the event of whether or not a vehicle is equipped is random with the probability of the market penetration rate, denoted by μ ($\nu = 1 - \mu$).

Here all events associated with vehicles in a traffic stream are assumed to be independently and identically distributed (iid) and form Bernoulli trials.

Further, we assume that information propagates in the fashion of "most forwarded within range" (MFR) (Takagi and Kleinrock, 1984). That is, a piece of information is transmitted to the farthest equipped vehicle within the transmission range of a sender. This assumption does not affect the computation of connectivity between two equipped vehicles, since they are connected as long as there exists an MFR communication path in between. In reality, however, information propagation does not have to follow the MFR manner, although it is the fastest approach to sending a message.

2.2 Definitions

For a general traffic stream, stationary under assumption, we denote the information source by vehicle 0 and the following vehicles in the direction of information flow in order by 1, 2, $\cdots$. Assuming that no two vehicles have the same location (even on a multi-lane road), we then have a unique position $x(k)$ for vehicle k [1]. Here we assume that $x(k+1) \geq x(k)$. That is, information propagates in the positive direction. We can obtain similar results when information propagates in the negative direction.

Within the transmission range R of information source at $x(0)$, we can find the farthest vehicle n_1 and denote the region $(x(0), x(n_1)]$ by *cell* 1 [2]. In a similar fashion, we find the farthest vehicle n_2 within the transmission range of vehicle n_1 and obtain cell 2 as $(x(n_1), x(n_2)]$. Repeating this process, we can divide a traffic stream into a number of cells, with a sequence of boundary vehicles $n_0 = 0$, n_1, $\cdots$, where $x(n_c) \leq cR$. We denote the cell where vehicle k is located by $C(k)$. Therefore, $C(k) = c$ if and only if $x(n_{c-1}) < x(k) \leq x(n_c)$.

[1] Although the existence of two vehicles of the same location could affect the implementation of certain communication protocols, it does not affect the connectivity significantly.

[2] A cell is called a cycle in (Cheng and Robertazzi, 1989), as shown in Fig. 1 therein.

Further, we define the *farthest upstream and downstream* (in the direction of information propagation) vehicles within the transmission range of vehicle $k > 0$ as $\underline{k}$ and $\overline{k}$, respectively. That is, $\underline{k} \geq 0$ is the minimum k satisfying $|x(k) - x(\underline{k})| \leq R$, and $\overline{k}$ is the maximum k satisfying $|x(\overline{k}) - x(k)| \leq R$. The farthest upstream and downstream vehicles of a vehicle define the boundary of a transmission and can be used to tell whether a vehicle is within its transmission range or not. Without loss of generality, we only consider a *continuous traffic stream* of K vehicles (excluding the information source) in the sense that no gap between two consecutive vehicles is bigger than the transmission range R. Therefore, in a continuous traffic stream, $\overline{k} > k$ for all $k = 0, \cdots, K-1$. That is, a vehicle except the last one always has a downstream vehicle within its transmission range. In such a traffic stream, we have the following observations:

1. When $C(k) = 1$, $\underline{k} = 0$. I.e., the farthest upstream vehicle of any vehicle in cell 1 is the information source.

2. When $C(k) = C(K)$, $\overline{k} = K$. I.e., the farthest downstream vehicle of any vehicle in the last cell is vehicle K.

3. $C(\underline{k}) = C(k) - 1$ when $0 < C(k) \leq C(K)$. I.e., the farthest upstream vehicles of all vehicles except the information source are in the preceding cell.

4. $C(\overline{k}) = C(k) + 1$ when $0 \leq C(k) < C(K)$. I.e., the farthest downstream vehicles of all vehicles except those in the last cell are in the following cell. In the last cell, $C(\overline{k}) = C(k) = C(K)$.

For any realization of Bernoulli trials of equipped vehicles, positions of all equipped vehicles are given. Thus, starting from the information source (denoted by node 0), we can repeat the process of finding the farthest equipped vehicle within the communication range of the current node as the next *node*, and these nodes form an MFR *communication chain*, or simply a communication chain. In a communication chain, the connection between two consecutive nodes is called a *hop*. For detailed regulatory properties of nodes and hops, please refer to (Jin and Recker, 2006).

3 An analytical model of multihop connectivity of an IVC system in a traffic stream

We denote the event when vehicle k is node h on a communication chain by $(k;h)$, the event when vehicle k is node h and also the end node by $(k;\bar{h})$, and the joint event of $(j;h-1)$ and $(k;h)$ by $(j,k;h)$. The corresponding probabilities are denoted by $P(k;h)$, $\bar{P}(k;h)$, and $\mathcal{P}(j,k;h)$, respectively. Hereafter we refer to $P(k;h)$ by *node probability*, $\bar{P}(k;h)$ by *end-node probability*, and $\mathcal{P}(j,k;h)$ by *hop probability*, since $\mathcal{P}(j,k;h)$ is the probability for the connection between vehicles j and k to be the hth hop. In the following subsections, we first discuss some basic properties for the three events and their corresponding probabilities and then present some fundamental relationships between them, which are applied to compute all probabilities and multihop connectivity of an IVC system.

3.1 Properties of and relationships between hop and node probabilities

For the three basic events and their corresponding probabilities, their properties are given by the following theorem. Here the results are quite straightforward, and their proofs omitted.

Theorem 3.1 (Basic properties of hop and node probabilities) *For a one-dimensional IVC system defined in the preceding section and the aforementioned probabilities, we can observe the following properties:*

1. *Event $(k;h)$ can only occur when $C(k) \leq h \leq 2C(k)-1$, since there can be only one node in cell 1 and one or two nodes in other cells (Jin and Recker, 2006). That is, $P(k;h)=0$ if $h < C(k)$, or $h > 2C(k)-1$. Obviously $P(0;0)=1$, since the information source is always node 0 by definition.*

2. *$\mathcal{P}(j,k;h)=0$ when $j < \underline{k}$ or $k > \overline{j}$; i.e., when vehicles j and k are outside of the transmission range of each other.*

3. *Since event $(j,k;h)$ is the joint event of $(j;h-1)$ and $(k;h)$, $\mathcal{P}(j,k;h) \leq P(j,h-1)$ and $\mathcal{P}(j,k;h) \leq P(k;h)$. However, $\mathcal{P}(j,k;h)$ may not equal $P(j;h-1)P(k;h)$, since the latter two events may not be independent.*

4. *$\mathcal{P}(j,k;h)=0$ when $h < C(k)$ or $h > 2C(k)-1$. Similarly, $\mathcal{P}(j,k;h)=0$ when $h-1 < C(j)$ or $h-1 > 2C(j)-1$.*

5. *Events $(j,k;h)$ and $(i,k;h)$ are mutually exclusive when $i \neq j$, since in any Bernoulli trials node $(k;h)$ can have only one preceding node.*

6. *For vehicle k in cell 1, i.e., $C(k)=1$, $\mathcal{P}(j,k;h)=0$ for $j=1,\cdots,k-1$, since vehicle k is within the transmission range of the information source and there cannot be an intermediate node j.*

Associated with any vehicle $k > 0$, there are the following probabilities, where $h \in [C(k), 2C(k)-1]$, $j \in [\underline{k}, k-1]$, and $l \in [k+1, \overline{k}]$: $P(k;h)$, the probability for vehicle k to be node h of a communication chain; $\mathcal{P}(j,k;h)$, the probability for vehicle k to the end node of hop $h-1$; $\mathcal{P}(k,l;h+1)$, the probability for vehicle k to be the start node of hop h. In the following theorem, we establish the relationships between these probabilities.

Theorem 3.2 (Fundamental relationships between node and hop probabilities) *Node* *probabilities can be computed from hop probabilities by*

$$P(k;h) \;=\; \sum_{j=\underline{k}}^{k-1} \mathcal{P}(j,k;h). \tag{1}$$

Hop probabilities can be computed in a recursive fashion as follows

$$\mathcal{P}(k,l;h+1) \;=\; \left(P(k;h) - \sum_{j=\underline{l}}^{k-1} \mathcal{P}(j,k;h)\right) \mu\nu^{\overline{k}-l}. \tag{2}$$

In addition, end-node probabilities can be computed from node probabilities and hop probabilities as follows:

$$\bar{P}(k;h) = P(k;h) - \sum_{l=k+1}^{\overline{k}} \mathcal{P}(k,l;h+1). \tag{3}$$

Proof. First, since vehicle $(k;h)$ must be the end of a hop from a vehicle in the front, e.g., vehicle $(j;h-1)$, event $(k;h)$ is a union of all joint events $(j,k;h)$ for all $j = \underline{k}, \cdots, k-1$. Further, since $(i,k;h)$ and $(j,k;h)$ are mutually exclusive when $i \neq j$ according to Theorem 3.1, we can have the node probability for $(k;h)$ as given in (1).

Second, we derive the recursive model, (2), for computing hop probability $\mathcal{P}(k,l;h+1)$ ($k < l \leq \overline{k}$ and $C(k) \leq h \leq 2C(k)-1$)). Event $(k,l;h+1)$ occurs if and only if the following conditions are satisfied: (i) Vehicle k is node h, but vehicles $\underline{l}$ to $k-1$ cannot be node $h-1$ due to the assumption of MFR; (ii) Vehicle l is equipped; (iii) Vehicles $l+1$ to $\overline{k}$ cannot be equipped due to the assumption of MFR. Here the probability of event (i) is $P(k;h) - \sum_{j=\underline{l}}^{k-1} \mathcal{P}(j,k;h)$, that of event (ii) is μ, and that of event (iii) is $\nu^{\overline{k}-l}$. Since the three events are all independent of each other, we have the probability of the joint events $\mathcal{P}(k,l;h+1)$ as given in (2).

Finally, if vehicle k is node h and the end node of a communication chain, then no vehicle between $k+1$ and $\overline{k}$ can be node $h+1$. That is, the probability $\bar{P}(k;h)$ is equal to $P(k;h)$ minus the probability for k to be the start of $h+1$th hop. Therefore we can obtain (3). ■

With the fundamental relationships between the probabilities, we are able to recursively compute these probabilities in a finite, continuous traffic stream with K vehicles in addition to the information source. First, for the information source $k = 0$ and $C(k) = 0$, we know that $P(0; 0) = 1$ and can compute $P(0, k; 1)$ with $C(k) = 1$ from (2) as

$$\mathcal{P}(0, k; 1) = \mu\nu^{n_1-k}, \tag{4}$$

since $\underline{k} = 0 > -1$. From (3) we can then compute $\bar{P}(0; 0)$. Second for $k = 1$ and $C(k) = 1$, we have from (1) that $P(k; 1) = \mathcal{P}(0, k; 1) = \mu\nu^{n_1-k}$, and from (2) that $\mathcal{P}(k, l; 2) = \left(P(k; 1) - \sum_{j=\underline{l}}^{k-1} \mathcal{P}(j, k; 1)\right) \mu\nu^{\overline{k}-l}$. If $C(l) = 1$, $\underline{l} = 0$, and $\mathcal{P}(k, l; 2) = (P(k; 1) - \mathcal{P}(0, k; 1))\, \mu\nu^{\overline{k}-l} = 0$. This is consistent with the observation in Theorem 3.1. Then from (3) we can compute the end node probability $\bar{P}(k; 1)$. Further, assuming that all probabilities are known up to vehicle k, we can then compute the corresponding quantities for vehicle $k+1$ by

$$\begin{aligned}
P(k+1; h) &= \sum_{j=\underline{k+1}}^{k} \mathcal{P}(j, k+1; h), \\
\mathcal{P}(k+1, l; h+1) &= \left(P(k+1; h) - \sum_{j=\underline{l+1}}^{k} \mathcal{P}(j, k; h)\right) \mu\nu^{\overline{k+1}-l}, \quad l \in [k+2, \overline{k+1}] \\
\bar{P}(k+1; h) &= P(k+1; h) - \sum_{l=k+2}^{\overline{k+1}} \mathcal{P}(k+1, l; h+1),
\end{aligned}$$

where $h \in [C(k+1), 2C(k+1) - 1]$.

In particular, for vehicle k in cell 1, i.e., $C(k) = 1$, we can have node probabilities for vehicles in cell 1 as

$$P(k; 1) = \mu\nu^{n_1-k}. \tag{5}$$

This can also be explained as follows: if vehicle k is node 1, it is the farthest equipped within the transmission range of the information source. Therefore, (i) vehicle k is equipped (with probability μ), and (ii) vehicles $k+1$ to n_1 are not (with probability ν^{n_1-k}). Then we have a geometric distribution of node probability in (5).

3.2 Performance measures of connectivity

From the end-node probability $\bar{P}(k;h)$, we can define the following performance measures of connectivity, with vehicle K as the last vehicle in a traffic stream.

1. The probability for information to stop at k, regardless of the number of hops, is

$$\bar{P}(k) = \sum_{h=C(k)}^{2C(k)-1} \bar{P}(k;h),$$

   which is also the probability for $x(k)$ to be the longest propagation distance.

2. The probability for information to travel to or beyond k, regardless of the number of hops, is

$$s(k) = \sum_{i=k}^{K} \bar{P}(i).$$

3. The probability for information to stop at h hops, regardless of the final position, is

$$\bar{P}(h) = \sum_{k=1}^{K} \bar{P}(k;h).$$

4. The expected number of hops is

$$E(h) = \sum_{h=1}^{\infty} h\bar{P}(h).$$

5. The expected information propagation distance is

$$E(x) = \sum_{k=1}^{K} x(k)\bar{P}(k),$$

   which is the mean of longest propagation distance used in (Yang and Recker, 2005).

6. The connectivity between information source and an equipped vehicle k is

$$\bar{s}(k) = s(\underline{k}). \tag{6}$$

That is, equipped vehicle k receives information whenever its farthest upstream vehicle $\underline{k}$ does. Here the connectivity $\bar{s}(k)$ can also be defined for road-side stations, as long as they are connected through IVC.

7. The s_0-connectivity front is $x_{s_0} = \{x(k)|\bar{s}(k) = s_0\}$.

8. The s_0-connectivity distance equals the distance between s_0-connectivity front and the information source, $x_{s_0} - x(0)$.

Unlike the existing models in the literature, the performance measures based on the model developed here can capture hop-related information, which is helpful in determining the fidelity of each hop or when a routing protocol has a limitation on the number of hops (Briesemeister et al., 2000). Moreover, in contrast to the lower bound obtained in (Jin and Recker, 2006), $s(k)$ is the absolute success rate. In the examples that follow, if not specifically noted otherwise, we express the multihop connectivity by $\bar{s}(k)$ in (6), which was also used in (Hartenstein et al., 2001). Note that $\bar{s}(k) = 1$ when $C(k) = 1$; i.e., if an equipped vehicle is in cell 1, it will be informed.

4 Examples

In this section, the positions of vehicles are predefined or computed from traffic flow models. That is, at any time instant, we have $x(0)$, $x(1)$, $\cdots$, $x(K)$ for a traffic stream with K vehicles. From the definitions in Section 2.2, we can obtain for a vehicle its cell-membership, and farthest upstream and downstream vehicles. Then from the model in Section 3.1, we can compute all probabilities starting from the first vehicle to the last. Further, from definitions in Section 3.2, we can compute the performance measures that we are interested in.

4.1 Uniform traffic

We first present results on multihop connectivity of a uniform traffic stream, in which gaps between vehicles are uniform, for different densities ρ and communication ranges R, assuming a penetration rate μ=10%.

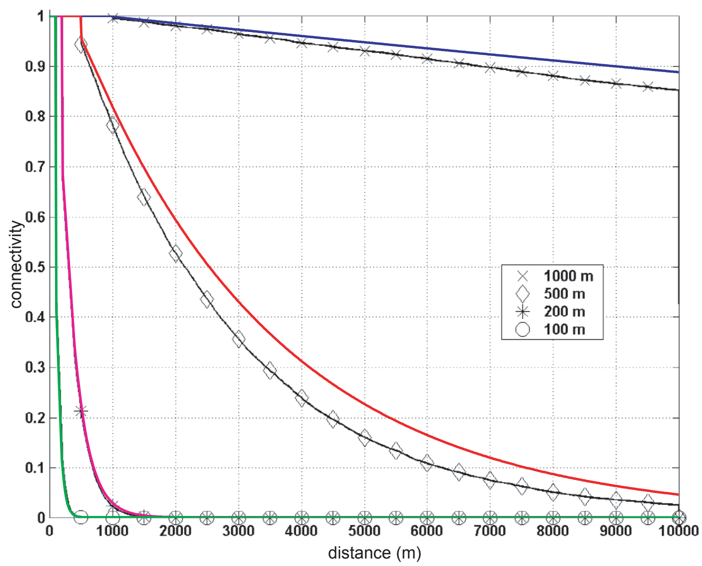

Figure 1: Multihop connectivity in a traffic stream with density=58 veh/km and market penetration rate=10%

In Figure 1, the connectivity between an equipped vehicle and the information source is shown for a uniform traffic stream with density of 58 veh/km, for four transmission ranges R=1, 0.5, 0.2, 0.1 km. In the figure, lines with marks are from Figure 1 of (Hartenstein et al., 2001) [3], and the corresponding solid lines are from the analytical model developed in the preceding section. As expected, the connectivity probabilities are higher than the lower bounds found in (Jin and Recker, 2006), and the connectivity increases with transmis-

[3]Figure 1 of (Hartenstein et al., 2001) considered a traffic stream on 2×2 lanes with average distances between two vehicles at 69m. That is, the total density is approximately $58 \approx 1000/69 \cdot 4$.

sion range. In addition, the analytical model yields results highly consistent with those in (Hartenstein et al., 2001). Since in (Hartenstein et al., 2001) traffic dynamics was considered, we can see that vehicle mobility has limited impact on multihop connectivity in this case. This observation is also supported by results in Figure 2, which compares Figure 2 of (Hartenstein et al., 2001) [4] with the analytical results of instantaneous multihop connectivity for a sparser traffic stream with density of 19 veh/km. Comparing the two figures, we can see that lower traffic density yields worse connectivity, as expected.

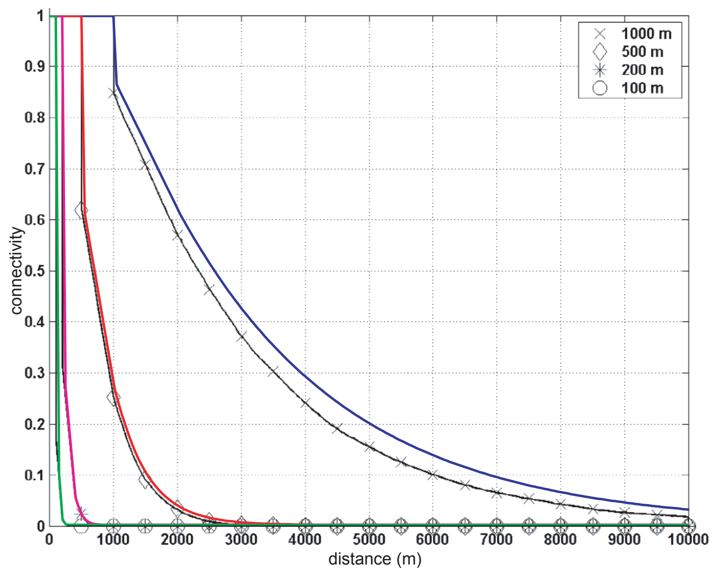

Figure 2: Multihop connectivity in a traffic stream with density=19 veh/km and market penetration rate=10%

[4]Figure 2 of (Hartenstein et al., 2001) considered a traffic stream on 2×2 lanes with average distances between two vehicles at 208m. That is, the total density is approximately $19 \approx 1000/208 \cdot 4$.

4.2 Traffic stream with a shock wave

We next consider a non-uniform traffic stream on a two-lane road. Initially at time $t = 0$, traffic density is $\rho_- = 30$ veh/km/lane and $\rho_+ = 40$ veh/km/lane for traffic upstream and downstream of $x = 0$, respectively. We assume that $\mu = 0.10$ and $R = 1$ km, and initially information is carried by a vehicle at $x_0 < 0$. We are interested in the instantaneous multihop connectivity in the IVC system. That is, at any time instant, we take a snapshot of all vehicles on the road and then obtain multihop connectivity and 95%-connectivity front.

Here vehicle positions at a time instant are predicted by a simple kinematic wave theory of vehicular traffic (Lighthill and Whitham, 1955; Richards, 1956), which incorporates a density-speed relationship, $v = V(\rho)$ (Hall et al., 1986). In this example, we use the following triangular fundamental diagram (Munjal et al., 1971; Haberman, 1977; Newell, 1993),

$$V(\rho) = \begin{cases} v_f, & 0 \leq \rho \leq \rho_c; \\ \frac{\rho_c}{\rho_j - \rho_c} \frac{\rho_j - \rho}{\rho} v_f, & \rho_c < \rho \leq \rho_j, \end{cases} \tag{7}$$

where v_f=104 km/h is the free flow speed, ρ_j=150 veh/km/lane the jam density, and $\rho_c = 0.2\rho_j$=30 veh/km/lane the critical density at which flow-rate, $q = \rho v$, attains its maximum — the capacity. According to the kinematic wave theory, a shock wave forms, and the shock wave interface, initially at $x = 0$, travels backward at constant speed $v_s = -26$ km/h. From the triangular fundamental diagram, vehicles upstream to the shock wave interface travel at $v_- = 104$ km/h, and those downstream travel at $v_+ = 71.5$ km/h. Then given a vehicle's initial location, we can obtain its location at any time.

With exact locations of all vehicles, including the information source, we can compute multihop connectivity at any time instant between the information source and any vehicle. In Figure 3(a), we show the connectivity between the information source and vehicles in both its forward and backward directions for four time instants: $t_0 = 0$, $t_1 = 2.3$ min, $t_2 = 4.6$ min, and t_3=11.7 min. The corresponding locations of the information source are -10 km,

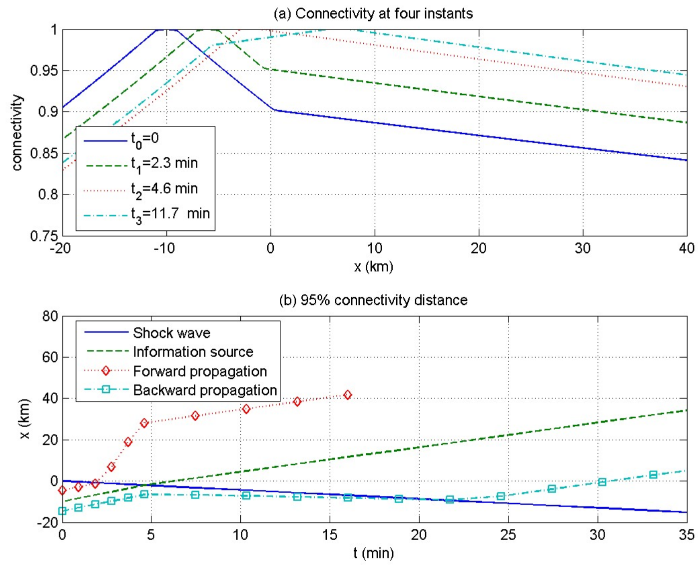

Figure 3: Instantaneous connectivity in traffic with a moving shock wave

-6 km, -2 km, and 6.5 km, respectively, and the locations of the shock wave interface are 0, -1 km, -2 km, and -5.1 km, respectively. Note that the information source meets the shock wave interface at t_2. In the figure, the curve for each time instant comprises a forward and a backward branch. The slope of the forward branches at t_0 and t_1 becomes smaller after meeting the shock wave, since the downstream of the shock wave has higher density. In the backward branches at t_3, the slope becomes larger upstream of the shock wave. In Figure 3(b), we show the positions of the shock wave interface, the information source, and the 95%-connectivity front in both forward and backward directions from $t = 0$ to $t = 35$ min. From the changing patterns of both forward and backward 95%-connectivity fronts, we can see that information propagation differs significantly in the upstream (sparser) and the downstream (denser) traffic to the shock wave. From these results, we can see that multihop connectivity between two equipped vehicles through inter-vehicle communication can be significantly affected by the distribution pattern of vehicles in non-uniform traffic.

5 Conclusion

In this paper, we developed a recursive model for computing end node probabilities, from which various performance measures of multihop connectivity were defined. The model assumes instantaneous information propagation, a condition that applies to situations for which market penetration is sufficiently high so as to minimize the impact of vehicle movement. We then studied examples of multihop connectivity in both uniform and non-uniform traffic and showed that the distribution patterns of vehicles on a road can significantly affect the performance of IVC. This study applies the same concepts of communication chains as (Jin and Recker, 2006). However, here we introduce end node probabilities, from which we can define accurate multihop connectivity between two nodes. The validity of the multihop connectivity model has been verified by Monte Carlo simulations in (Jin and Recker, 2007). The model can be used for any 1-D mobile ad hoc network with arbitrary distribution patterns of communication nodes. It is more general than existing models that assume spatial Poisson distribution of communication nodes and can easily be generalized to incorporate such fidelity factors as failure at each hop.

The performance measures of connectivity proposed in this paper and the analysis procedures developed could have direct implications for many transportation applications based on IVC, including IVC-based incident response systems, whose success hinges upon the coverage of an incident message. The results of this study serve to better understand how well information may propagate through IVC under certain conditions, and to help determine properties of communication devices (in terms of communication range) that potentially could be used to achieve certain performance of such systems.

The models developed in this study can easily be extended to help to determine the distance between consecutive road-side stations, deployed as a supplement to the IVC system, to relay information to vehicles in order to achieve better success rates (Jin and Wang, 2008).

Another extension to this work involves developing models for information propagation in dynamically changing traffic streams. We believe that such a problem can be tackled in a similar fashion since, compared to other mobile ad hoc networks (Perkins, 2000), the topology change caused by traffic dynamics is relatively small. It would also be desirable to develop an information propagation model coupled with a traffic flow model, such as the Lighthill-Whitham-Richards model (Lighthill and Whitham, 1955; Richards, 1956) and consider information propagation for different road networks. Finally, the analysis methods developed in this study could be extended to obtain the accurate connectivity on two-dimensional road networks, which is still an open problem (Tang et al., 2003).

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